

## DAY FOUR

# Quadratic Equation and Inequalities

### Learning & Revision for the Day

- Quadratic Equation
- Relation between Roots and Coefficients
- Formation of an Equation
- Transformation of Equations
- Maximum and Minimum Value of  $ax^2 + bx + c$
- Sign of Quadratic Expression
- Position of Roots
- Inequalities
- Arithmetic-Geometric-Harmonic Mean Inequality
- Logarithm Inequality

## Quadratic Equation

- An equation of the form  $ax^2 + bx + c = 0$ , where  $a \neq 0$ ,  $a$ ,  $b$  and  $c$ ,  $x \in R$ , is called a **real quadratic equation**. Here  $a$ ,  $b$  and  $c$  are called the **coefficients** of the equation.
- The quantity  $D = b^2 - 4ac$  is known as the **discriminant** of the equation  $ax^2 + bx + c = 0$  and its roots are given by  $x = \frac{-b \pm \sqrt{D}}{2a}$
- An equation of the form  $az^2 + bz + c = 0$ , where  $a \neq 0$ ,  $a$ ,  $b$  and  $c$ ,  $z \in C$  (complex) is called a **complex quadratic equation** and its roots are given by  $z = \frac{-b \pm \sqrt{D}}{2a}$ .

## Nature of Roots of Quadratic Equation

Let  $a, b, c \in R$  and  $a \neq 0$ , then the equation  $ax^2 + bx + c = 0$

- (i) has real and distinct roots if and only if  $D > 0$ .
- (ii) has real and equal roots if and only if  $D = 0$ .
- (iii) has complex roots with non-zero imaginary parts if and only if  $D < 0$ .



## Some Important Results

- (i) If  $p + iq$  (where,  $p, q \in R, q \neq 0$ ) is one root of  $ax^2 + bx + c = 0$ , then second root will be  $p - iq$
- (ii) If  $a, b, c \in Q$  and  $p + \sqrt{q}$  is an **irrational root** of  $ax^2 + bx + c = 0$ , then other root will be  $p - \sqrt{q}$ .
- (iii) If  $a, b, c \in Q$  and  $D$  is a perfect square, then  $ax^2 + bx + c = 0$  has **rational roots**.
- (iv) If  $a = 1, b, c \in I$  and roots of  $ax^2 + bx + c = 0$  are rational numbers, then these roots must be integers.
- (v) If the roots of  $ax^2 + bx + c = 0$  are both positive, then the signs of  $a$  and  $c$  should be like and opposite to the sign of  $b$ .
- (vi) If the roots of  $ax^2 + bx + c = 0$  are both negative, then signs of  $a, b$  and  $c$  should be like.
- (vii) If the roots of  $ax^2 + bx + c = 0$  are reciprocal to each other, then  $c = a$ .
- (viii) In the equation  $ax^2 + bx + c = 0$  ( $a, b, c \in R$ ), if  $a + b + c = 0$ , then the roots are  $1, \frac{c}{a}$  and if  $a - b + c = 0$ , then the roots are  $-1$  and  $-\frac{c}{a}$ .

## Relation between Roots and Coefficients

### Quadratic Roots

If  $\alpha$  and  $\beta$  are the roots of quadratic equation  $ax^2 + bx + c = 0$ ;

$a \neq 0$ , then sum of roots  $= \alpha + \beta = -\frac{b}{a}$

and product of roots  $= \alpha\beta = \frac{c}{a}$ .

And, also  $ax^2 + bx + c = a(x - \alpha)(x - \beta)$

### Cubic Roots

If  $\alpha, \beta$  and  $\gamma$  are the roots of cubic equation

$ax^3 + bx^2 + cx + d = 0$ ;  $a \neq 0$ , then  $\alpha + \beta + \gamma = -\frac{b}{a}$

$$\beta\gamma + \gamma\alpha + \alpha\beta = \frac{c}{a}$$

and

$$\alpha\beta\gamma = -\frac{d}{a}$$

### Common Roots (Conditions)

Suppose that the quadratic equations are  $ax^2 + bx + c = 0$  and  $a'x^2 + b'x + c' = 0$ .

- (i) When **one root** is common, then the condition is  $(a'c - ac')^2 = (bc' - b'c)(ab' - a'b)$ .
- (ii) When **both roots** are common, then the condition is  $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$ .

## Formation of an Equation

### Quadratic Equation

If the roots of a quadratic equation are  $\alpha$  and  $\beta$ , then the equation will be of the form  $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ .

### Cubic Equation

If  $\alpha, \beta$  and  $\gamma$  are the roots of the cubic equation, then the equation will be form of

$$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0.$$

## Transformation of Equations

Let the given equation be

$$a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n = 0 \quad \dots(A)$$

Then, the equation

- (i) whose roots are  $k$  ( $\neq 0$ ) times roots of the Eq. (A), is obtained by replacing  $x$  by  $\frac{x}{k}$  in Eq. (A).
- (ii) whose roots are the negatives of the roots of Eq. (A), is obtained by replacing  $x$  by  $-x$  in Eq. (A).
- (iii) whose roots are  $k$  more than the roots of Eq. (A), is obtained by replacing  $x$  by  $(x - k)$  in Eq. (A).
- (iv) whose roots are reciprocals of the roots of Eq. (A), is obtained by replacing  $x$  by  $1/x$  in Eq. (A) and then multiply both the sides by  $x^n$ .

## Maximum and Minimum Value of $ax^2 + bx + c$

- (i) When  $a > 0$ , then minimum value of  $ax^2 + bx + c$  is

$$\frac{-D}{4a} \text{ or } \frac{4ac - b^2}{4a} \text{ at } x = \frac{-b}{2a}$$

- (ii) When  $a < 0$ , then maximum value of  $ax^2 + bx + c$  is

$$\frac{-D}{4a} \text{ or } \frac{4ac - b^2}{4a} \text{ at } x = \frac{-b}{2a}$$

## Sign of Quadratic Expression

Let  $f(x) = ax^2 + bx + c$ , where  $a, b$  and  $c \in R$  and  $a \neq 0$ .

- (i) If  $a > 0$  and  $D < 0$ , then  $f(x) > 0, \forall x \in R$ .
- (ii) If  $a < 0$  and  $D < 0$ , then  $f(x) < 0, \forall x \in R$ .
- (iii) If  $a > 0$  and  $D = 0$ , then  $f(x) \geq 0, \forall x \in R$ .
- (iv) If  $a < 0$  and  $D = 0$ , then  $f(x) \leq 0, \forall x \in R$ .
- (v) If  $a > 0, D > 0$  and  $f(x) = 0$  have two real roots  $\alpha$  and  $\beta$ , where  $(\alpha < \beta)$ , then  $f(x) > 0, \forall x \in (-\infty, \alpha) \cup (\beta, \infty)$  and  $f(x) < 0, \forall x \in (\alpha, \beta)$ .
- (vi) If  $a < 0, D > 0$  and  $f(x) = 0$  have two real roots  $\alpha$  and  $\beta$ , where  $(\alpha < \beta)$ , then  $f(x) < 0, \forall x \in (-\infty, \alpha) \cup (\beta, \infty)$  and  $f(x) > 0, \forall x \in (\alpha, \beta)$ .

## Position of Roots

Let  $ax^2 + bx + c = 0$  has roots  $\alpha$  and  $\beta$ . Then, we have the following conditions:

- (i) with respect to one real number ( $k$ ).

Situation	Required conditions
$\alpha < \beta < k$	$D \geq 0, af(k) > 0, k > -b/2a$
$k < \alpha < \beta$	$D \geq 0, af(k) > 0, k < -b/2a$
$\alpha < k < \beta$	$D > 0, af(k) < 0$

- (ii) with respect to two real numbers  $k_1$  and  $k_2$ .

Situation	Required conditions
$k_1 < \alpha < \beta < k_2$	$D \geq 0, af(k_1) > 0,$ $af(k_2) > 0, k_1 < -b/2a < k_2$
$\alpha < k_1 < k_2 < \beta$	$D > 0, af(k_1) < 0, af(k_2) < 0$
$k_1 < \alpha < k_2 < \beta$	$D > 0, f(k_1)f(k_2) < 0$

## Inequalities

Let  $a$  and  $b$  be two real numbers. If  $a - b$  is negative, we say that  $a$  is less than  $b$  ( $a < b$ ) and if  $a - b$  is positive, then  $a$  is greater than  $b$  ( $a > b$ ). This shows the inequalities concept.

### Important Results on Inequalities

- (i) If  $a > b$ , then  $a \pm c > b \pm c, \forall c \in R$ .
- (ii) If  $a > b$ , then
- (a) for  $m > 0, am > bm, \frac{a}{m} > \frac{b}{m}$
- (b) for  $m < 0, am < bm, \frac{a}{m} < \frac{b}{m}$
- (iii) (a) If  $a > b > 0$ , then
- $a^2 > b^2$
  - $|a| > |b|$
  - $\frac{1}{a} < \frac{1}{b}$
- (b) If  $a < b < 0$ , then
- $a^2 > b^2$
  - $|a| > |b|$
  - $\frac{1}{a} > \frac{1}{b}$
- (iv) If  $a < 0 < b$ , then
- (a)  $a^2 > b^2$ , if  $|a| > |b|$  (b)  $a^2 < b^2$ , if  $|a| < |b|$
- (v) If  $a < x < b$  and  $a, b$  are positive real numbers, then  $a^2 < x^2 < b^2$ .
- (vi) If  $a < x < b$  and  $a$  is negative number and  $b$  is positive number, then
- (a)  $0 \leq x^2 < b^2$ , if  $|b| > |a|$  (b)  $0 \leq x^2 < a^2$ , if  $|a| > |b|$
- (vii) If  $a_i > b_i > 0$ , where  $i = 1, 2, 3, \dots, n$ , then  $a_1 a_2 a_3 \dots a_n > b_1 b_2 b_3 \dots b_n$ .
- (viii) If  $a_i > b_i$ , where  $i = 1, 2, 3, \dots, n$ , then  $a_1 + a_2 + a_3 + \dots + a_n > b_1 + b_2 + \dots + b_n$ .
- (ix) If  $|x| < a$ , then
- (a) for  $a > 0, -a < x < a$ .
- (b) for  $a < 0, x \in \phi$ .

## Arithmetic-Geometric-Harmonic Mean Inequality

The Arithmetic-Geometric-Harmonic Mean of positive real numbers is defined as follows

Arithmetic Mean  $\geq$  Geometric Mean  $\geq$  Harmonic Mean

(i) If  $a, b > 0$  then  $\frac{a+b}{2} \geq \sqrt{ab} \geq \frac{2}{\frac{1}{a} + \frac{1}{b}}$

- (ii) If  $a_i > 0$ , where  $i = 1, 2, 3, \dots, n$ , then

$$\frac{a_1 + a_2 + \dots + a_n}{n} \geq (a_1 \cdot a_2 \dots a_n)^{1/n} \geq \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n}}$$

## Logarithm Inequality

If  $a$  is a positive real number other than 1 and  $a^x = m$ , then  $x$  is called the logarithm of  $m$  to the base  $a$ , written as  $\log_a m$ . In  $\log_a m$ ,  $m$  should always be positive.

- (i) If  $m \leq 0$ , then  $\log_a m$  will be meaningless.
- (ii)  $\log_a m$  exists, if  $m, a > 0$  and  $a \neq 1$ .

### Important Results on Logarithm

- (i)  $a^{\log_a x} = x; a > 0, \neq 1, x > 0$
- (ii)  $a^{\log_b x} = x^{\log_b a}; a, b > 0, \neq 1, x > 0$
- (iii)  $\log_a a = 1, a > 0, \neq 1$
- (iv)  $\log_a x = \frac{1}{\log_x a}; x, a > 0, \neq 1$
- (v)  $\log_a x = \log_a b \log_b x = \frac{\log_b x}{\log_b a}; a, b > 0, \neq 1, x > 0$
- (vi) For  $x > 0; a > 0, \neq 1$
- (a)  $\log_{a^n}(x) = \frac{1}{n} \log_a x$
- (b)  $\log_{a^n} x^m = \left(\frac{m}{n}\right) \log_a x$
- (vii) For  $x > y > 0$
- (a)  $\log_a x > \log_a y$ , if  $a > 1$
- (b)  $\log_a x < \log_a y$ , if  $0 < a < 1$
- (viii) If  $a > 1$  and  $x > 0$ , then
- (a)  $\log_a x > p \Rightarrow x > a^p$
- (b)  $0 < \log_a x < p \Rightarrow 0 < x < a^p$
- (ix) If  $0 < a < 1$ , then
- (a)  $\log_a x > p \Rightarrow 0 < x < a^p$
- (b)  $0 < \log_a x < p \Rightarrow a^p < x < 1$

DAY PRACTICE SESSION 1

## FOUNDATION QUESTIONS EXERCISE

- 1 If  $\sqrt{3x^2 - 7x - 30} + \sqrt{2x^2 - 7x - 5} = x + 5$ , then  $x$  is equal to  
(a) 2      (b) 3      (c) 6      (d) 5
- 2 The number of solutions for equation  $x^2 - 5|x| + 6 = 0$  is  
(a) 4      (b) 3      (c) 2      (d) 1
- 3 The roots of the equation  $|2x - 1|^2 - 3|2x - 1| + 2 = 0$  are  
(a)  $\left\{-\frac{1}{2}, 0, \frac{1}{2}\right\}$  (b)  $\left\{-\frac{1}{2}, 0, \frac{3}{2}\right\}$  (c)  $\left\{-\frac{3}{2}, \frac{1}{2}, 0, 1\right\}$  (d)  $\left\{-\frac{1}{2}, 0, 1, \frac{3}{2}\right\}$
- 4 The product of all the values of  $x$  satisfying the equation  $(5 + 2\sqrt{6})^{x^2 - 3} + (5 - 2\sqrt{6})^{x^2 - 3} = 10$  is  
(a) 4      (b) 6      (c) 8      (d) 19
- 5 The root of the equation  $2(1+i)x^2 - 4(2-i)x - 5 - 3i = 0$ , where  $i = \sqrt{-1}$ , which has greater modulus, is  
(a)  $\frac{3-5i}{2}$       (b)  $\frac{5-3i}{2}$       (c)  $\frac{3+i}{2}$       (d)  $\frac{3i+1}{2}$
- 6  $x^2 + x + 1 + 2k(x^2 - x - 1) = 0$  is perfect square for how many value of  $k$   
(a) 2      (b) 0      (c) 1      (d) 3
- 7 If the roots of  $(a^2 + b^2)x^2 - 2(bc + ad)x + c^2 + d^2 = 0$  are equal, then  
(a)  $\frac{a}{b} = \frac{c}{d}$       (b)  $\frac{a}{c} + \frac{b}{d} = 0$       (c)  $\frac{a}{d} = \frac{b}{c}$       (d)  $a + b = c + d$
- 8 The least value of  $|\alpha|$  for which  $\tan \theta$  and  $\cot \theta$  are roots of the equation  $x^2 + ax + 1 = 0$ , is  
(a) 2      (b) 1      (c)  $1/2$       (d) 0
- 9 If one root of the equation  $x^2 - \lambda x + 12 = 0$  is even prime while  $x^2 + \lambda x + \mu = 0$  has equal roots, then  $\mu$  is equal to  
(a) 8      (b) 16      (c) 24      (d) 32
- 10 If  $a + b + c = 0$ , then the roots of the equation  $4ax^2 + 3bx + 2c = 0$ , where  $a, b, c \in R$  are  
(a) real and distinct      (b) imaginary  
(c) real and equal      (d) infinite
- 11 The equation  $(\cos \beta - 1)x^2 + (\cos \beta)x + \sin \beta = 0$  in the variable  $x$  has real roots, then  $\beta$  lies in the interval  
(a)  $(0, 2\pi)$       (b)  $(-\pi, 0)$       (c)  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$       (d)  $(0, \pi)$
- 12 If  $ax^2 + 2bx - 3c = 0$  has no real root and  $\frac{3c}{4} < a + b$ , then the range of  $c$  is  
(a)  $(-1, 1)$       (b)  $(0, 1)$   
(c)  $(0, \infty)$       (d)  $(-\infty, 0)$
- 13 If  $a, b$  and  $c$  are real numbers in AP, then the roots of  $ax^2 + bx + c = 0$  are real for  
(a) all  $a$  and  $c$       (b) no  $a$  and  $c$   
(c)  $\left|\frac{c}{a} - 7\right| \geq 4\sqrt{3}$       (d)  $\left|\frac{a}{c} + 7\right| \geq 2\sqrt{3}$
- 14 If  $P(x) = ax^2 + bx + c$  and  $Q(x) = -ax^2 + dx + c = 0$ ;  $ac \neq 0$ , then the equation  $P(x) \cdot Q(x) = 0$  has  
(a) four real roots      (b) exactly two real roots  
(c) either two or four real roots      (d) almost two real roots
- 15 The rational values of  $a$  in  $ax^2 + bx + 1 = 0$  if  $\frac{1}{4 + \sqrt{3}}$  is a root, are  
(a)  $a = 13, b = -8$       (b)  $a = -13, b = 8$   
(c)  $a = 13, b = 8$       (d)  $a = -13, b = -8$
- 16 If  $1 - i$ , is a root of the equation  $x^2 + ax + b = 0$ , where  $a, b \in R$ , then the values of  $a$  and  $b$  are  
(a) 1, -1      (b) 2, -2  
(c) 3, -3      (d) None of these
- 17 The values of  $p$  for which one root of the equation  $x^2 - 30x + p = 0$  is the square of the other, is/are  
(a) Only 125      (b) 125 and -216  
(c) 125 and 215      (d) Only 216
- 18 If the roots of the quadratic equation  $\frac{x-m}{mx+1} = \frac{x+n}{nx+1}$  are reciprocal to each other, then  
(a)  $n = 0$       (b)  $m = n$       (c)  $m + n = 1$       (d)  $m^2 + n^2 = 1$
- 19 Let  $\alpha$  and  $\alpha^2$  be the roots of  $x^2 + x + 1 = 0$ , then the equation whose roots are  $\alpha^{31}$  and  $\alpha^{62}$ , is  
(a)  $x^2 - x + 1 = 0$       (b)  $x^2 + x - 1 = 0$   
(c)  $x^2 + x + 1 = 0$       (d)  $x^{60} + x^{30} + 1 = 0$
- 20 If  $\alpha$  and  $\beta$  are the roots of  $x^2 - a(x-1) + b = 0$ , then the value of  $\frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} + \frac{2}{a+b}$  is  
(a)  $\frac{4}{a+b}$       (b)  $\frac{1}{a+b}$       (c) 0      (d) -1
- 21 The value of  $a$  for which the sum of the squares of the roots of the equation  $x^2 - (a-2)x - a - 1 = 0$  assume the least value is → AIEEE 2005  
(a) 2      (b) 3      (c) 0      (d) 1
- 22 If  $\alpha$  and  $\beta$  be the roots of the equation  $2x^2 + 2(a+b)x + a^2 + b^2 = 0$ , then the equation whose roots are  $(\alpha + \beta)^2$  and  $(\alpha - \beta)^2$ , is  
(a)  $x^2 - 2abx - (a^2 - b^2)^2 = 0$  (b)  $x^2 - 4abx - (a^2 - b^2)^2 = 0$   
(c)  $x^2 - 4abx + (a^2 - b^2)^2 = 0$  (d) None of these
- 23 Let  $\alpha, \beta$  be the roots of  $x^2 - 2x \cos \phi + 1 = 0$ , then the equation whose roots are  $\alpha^n$  and  $\beta^n$ , is  
(a)  $x^2 - 2x \cos n\phi - 1 = 0$       (b)  $x^2 - 2x \cos n\phi + 1 = 0$   
(c)  $x^2 - 2x \sin n\phi + 1 = 0$       (d)  $x^2 + 2x \sin n\phi - 1 = 0$
- 24 The harmonic mean of the roots of the equation  $(5 + \sqrt{2})x^2 - (4 + \sqrt{5})x + 8 + 2\sqrt{5} = 0$  is  
(a) 2      (b) 4      (c) 6      (d) 8

- 25** If the ratio of the roots of  $\lambda x^2 + \mu x + \nu = 0$  is equal to the ratio of the roots of  $x^2 + x + 1 = 0$ , then  $\lambda, \mu$  and  $\nu$  are in  
 (a) AP (b) GP  
 (c) HP (d) None of these
- 26** If the roots of the equation  $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$  are equal in magnitude but opposite in sign, then the product of the roots is  
 (a)  $-2(p^2 + q^2)$  (b)  $-(p^2 + q^2)$   
 (c)  $-\frac{(p^2 + q^2)}{2}$  (d)  $-pq$
- 27** If the roots of the equation  $ax^2 + bx + c = 0$  of the form  $\frac{k+1}{k}$  and  $\frac{k+2}{k+1}$ , then  $(a+b+c)^2$  is equal to  
 (a)  $2b^2 - ac$  (b)  $\Sigma a^2$  (c)  $b^2 - 4ac$  (d)  $b^2 - 2ac$
- 28** If  $\alpha$  and  $\beta$  are the roots of the equation  $ax^2 + bx + c = 0$  such that  $\beta < \alpha < 0$ , then the quadratic equation whose roots are  $|\alpha|, |\beta|$ , is given by  
 (a)  $|a|x^2 + |b|x + |c| = 0$  (b)  $ax^2 - |b|x + c = 0$   
 (c)  $|a|x^2 - |b|x + |c| = 0$  (d)  $a|x|^2 + b|x + c = 0$
- 29** If  $\alpha$  and  $\beta$  be the roots of  $x^2 + px + q = 0$ , then  $\frac{(\omega\alpha + \omega^2\beta)(\omega^2\alpha + \omega\beta)}{\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}}$  is equal to  
 (a)  $-\frac{q}{p}$  (b)  $\alpha\beta$  (c)  $-\frac{p}{q}$  (d)  $\omega$
- 30** If  $\alpha$  and  $\beta$  are roots of the equation  $x^2 + px + 3 \cdot \frac{p}{4} = 0$ , such that  $|\alpha - \beta| = \sqrt{10}$ , then  $p$  belongs **→ JEE Mains 2013**  
 (a)  $\{2, -5\}$  (b)  $\{-3, 2\}$  (c)  $\{-2, 5\}$  (d)  $\{3, -5\}$
- 31** Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots  $(4, 3)$ . Rahul made a mistake in writing down coefficient of  $x$  to get roots  $(3, 2)$ . The correct roots of equation are **→ AIEEE 2011**  
 (a)  $-4, -3$  (b)  $6, 1$  (c)  $4, 3$  (d)  $-6, -1$
- 32** If  $\alpha$  and  $\beta$  are the roots of  $x^2 + x + 2 = 0$  and  $\gamma, \delta$  are the roots of  $x^2 + 3x + 4 = 0$ , then  $(\alpha + \gamma)(\alpha + \delta)(\beta + \gamma)(\beta + \delta)$  is equal to  
 (a)  $-18$  (b)  $18$  (c)  $24$  (d)  $44$
- 33** In  $\Delta PQR, R = \frac{\pi}{2}$ . If  $\tan \frac{P}{2}$  and  $\tan \frac{Q}{2}$  are the roots of the equation  $ax^2 + bx + c = 0$ , then  
 (a)  $a = b + c$  (b)  $b = c + a$   
 (c)  $c = a + b$  (d)  $b = c$
- 34** If the equation  $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$  and  $6k(2x^2 + 1) + px + 4x^2 - 2 = 0$  have both roots common, then  $2r - p$  is equal to  
 (a)  $2$  (b)  $1$  (c)  $0$  (d)  $k$

- 35** If the equations  $x^2 + 2x + 3 = 0$  and  $ax^2 + bx + c = 0$ ,  $a, b, c \in R$ , have a common root, then  $a : b : c$  is **→ AIEEE 2012**  
 (a)  $1 : 2 : 3$  (b)  $3 : 2 : 1$  (c)  $1 : 3 : 2$  (d)  $3 : 1 : 2$
- 36** The equation formed by decreasing each root of  $ax^2 + bx + c = 0$  by 1 is  $2x^2 + 8x + 2 = 0$ , then  
 (a)  $a = -b$  (b)  $b = -c$  (c)  $c = -a$  (d)  $b = a + c$
- 37** If  $f(x) = x^2 + 2bx + 2c^2$  and  $g(x) = -x^2 - 2cx + b^2$  such that  $\min f(x) > \max g(x)$ , then the relation between  $b$  and  $c$  is  
 (a)  $|c| < |b| \sqrt{2}$  (b)  $0 < c < b \sqrt{2}$   
 (c)  $|c| < |b| 2$  (d)  $|c| > |b| \sqrt{2}$
- 38** If  $a \in R$  and  $a_1, a_2, a_3, \dots, a_n \in R$ , then  $(x - a_1)^2 + (x - a_2)^2 + \dots + (x - a_n)^2$  assumes its least value at  $x =$   
 (a)  $a_1 + a_2 + \dots + a_n$  (b)  $2(a_1 + a_2 + a_3 + \dots + a_n)$   
 (c)  $n(a_1 + a_2 + \dots + a_n)$  (d) None of these
- 39** If the roots of the equation  $bx^2 + cx + a = 0$  is imaginary, then for all real values of  $x$ , the expression  $3b^2x^2 + 6bcx + 2c^2$  is **→ AIEEE 2009**  
 (a) greater than  $4ab$  (b) less than  $4ab$   
 (c) greater than  $-4ab$  (d) less than  $-4ab$
- 40** If  $x^2 + 2ax + 10 - 3a > 0$  for all  $x \in R$ , then  
 (a)  $-5 < a < 2$  (b)  $a < -5$   
 (c)  $a > 5$  (d)  $2 < a < 5$
- 41** If the expression  $\left(ax - 1 + \frac{1}{x}\right)$  is non-negative for all positive real  $x$ , then the minimum value of  $a$  must be  
 (a)  $0$  (b)  $\frac{1}{2}$   
 (c)  $\frac{1}{4}$  (d) None of these
- 42** The number of real solutions of the equation  $\left(\frac{9}{10}\right)^x = -3 + x - x^2$  is  
 (a)  $0$  (b)  $1$   
 (c)  $2$  (d) None of these
- 43** If  $\alpha$  and  $\beta$  be the roots of the quadratic equation  $ax^2 + bx + c = 0$  and  $k$  be a real number, then the condition, so that  $\alpha < k < \beta$  is given by  
 (a)  $ac > 0$  (b)  $ak^2 + bk + c = 0$   
 (c)  $ac < 0$  (d)  $a^2k^2 + abk + ac < 0$
- 44**  $|2x - 3| < |x + 5|$ , then  $x$  belongs to  
 (a)  $(-3, 5)$  (b)  $(5, 9)$  (c)  $\left(-\frac{2}{3}, 8\right)$  (d)  $\left(-8, \frac{2}{3}\right)$
- 45** The least integral value  $\alpha$  of  $x$  such that  $\frac{x-5}{x^2+5x-14} > 0$ , satisfies **→ JEE Mains 2013**  
 (a)  $\alpha^2 + 3\alpha - 4 = 0$  (b)  $\alpha^2 - 5\alpha + 4 = 0$   
 (c)  $\alpha^2 - 7\alpha + 6 = 0$  (d)  $\alpha^2 + 5\alpha - 6 = 0$

- 46 The solution set of  $\frac{|x-2|-1}{|x-2|-2} \leq 0$  is  
 (a)  $[0, 1] \cup (3, 4)$  (b)  $[0, 1] \cup [3, 4]$   
 (c)  $[-1, 1) \cup (3, 4]$  (d) None of these
- 47 Number of integral solutions of  $\frac{x+2}{x^2+1} > \frac{1}{2}$  is  
 (a) 0 (b) 1 (c) 2 (d) 3
- 48 If the product of  $n$  positive numbers is 1, then their sum is  
 (a) a positive integer (b) divisible by  $n$   
 (c) equation to  $n + \frac{1}{n}$  (d) greater than or equal to  $n$
- 49 The minimum value of  $P = bcx + cay + abz$ , when  $xyz = abc$ , is  
 (a)  $3abc$  (b)  $6abc$   
 (c)  $abc$  (d)  $4abc$
- 50 If  $a, b$  and  $c$  are distinct three positive real numbers, then  $(a+b+c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$  is  
 (a)  $> 9$  (b)  $> 9$   
 (c)  $< 9$  (d) None of these

- 51 If  $a, b, c$  are positive real numbers such that  $a + b + c = 1$ , then the greatest value of  $(1-a)(1-b)(1-c)$ , is  
 (a)  $\frac{1}{27}$  (b)  $\frac{8}{27}$  (c)  $\frac{4}{27}$  (d) 9
- 52 If  $a, b, c, d$  are positive real numbers such that  $a + b + c + d = 2$ , then  $M = (a+b)(c+d)$  satisfies the relation  
 (a)  $0 \leq M \leq 1$  (b)  $1 \leq M \leq 2$  (c)  $2 \leq M \leq 3$  (d)  $3 \leq M \leq 4$
- 53  $\log_2(x^2 - 3x + 18) < 4$ , then  $x$  belongs to  
 (a) (1, 2) (b) (2, 16)  
 (c) (1, 16) (d) None of these
- 54 If  $\log_{0.3}(x-1) > \log_{0.09}(x-1)$ , then  $x$  lies in  
 (a) (1, 2) (b)  $(-\infty, 1)$   
 (c) (2,  $\infty$ ) (d) None of these
- 55 What is the solution set of the following inequality?

$$\log_x\left(\frac{x+5}{1-3x}\right) > 0$$

- (a)  $0 < x < \frac{1}{3}$  (b)  $x \geq 3$   
 (c)  $\frac{1}{3} < x < 1$  (d) None of these

## DAY PRACTICE SESSION 2

# PROGRESSIVE QUESTIONS EXERCISE

- 1 Let  $S = \{x \in R : x \geq 0 \text{ and } 2\sqrt{x} - 3 + \sqrt{x}(\sqrt{x} - 6) + 6 = 0$   
 Then,  $S$  → JEE Mains 2018  
 (a) is an empty set  
 (b) contains exactly one element  
 (c) contains exactly two elements  
 (d) contains exactly four elements
- 2 The roots of the equation  $2^{x+2} \cdot 3^{3x/(x-1)} = 9$  are given by  
 (a)  $1 - \log_2 3, 2$  (b)  $\log_2\left(\frac{2}{3}\right), 1$   
 (c)  $2, -2$  (d)  $-2, 1 - \frac{(\log 3)}{(\log 2)}$
- 3 Let  $\alpha$  and  $\beta$  be the roots equation  $x^2 - 6x - 2 = 0$ . If  $a_n = \alpha^n - \beta^n$  for  $n \geq 1$ , then the value of  $\frac{a_{10} - 2a_8}{2a_9}$  is equal to  
 → JEE Mains 2015  
 (a) 6 (b) -6 (c) 3 (d) -3
- 4 If  $x^2 - 5x + 1 = 0$ , then  $\frac{x^{10} + 1}{x^5}$  is equal to  
 (a) 2424 (b) 3232  
 (c) 2525 (d) None of these
- 5 If  $a < b < c < d$ , then the roots of the equation  $(x-a)(x-c) + 2(x-b)(x-d) = 0$  are  
 (a) real and distinct (b) real and equal  
 (c) imaginary (d) None of these
- 6 Let  $\alpha$  and  $\beta$  be the roots of equation  $px^2 + qx + r = 0$ ,  $p \neq 0$ . If  $p, q$  and  $r$  are in AP and  $\frac{1}{\alpha} + \frac{1}{\beta} = 4$ , then the value of  $|\alpha - \beta|$  is → JEE Mains 2014  
 (a)  $\frac{\sqrt{61}}{9}$  (b)  $\frac{2\sqrt{17}}{9}$  (c)  $\frac{\sqrt{34}}{9}$  (d)  $\frac{2\sqrt{13}}{9}$
- 7 If  $\alpha$  and  $\beta$  are the roots of the equation  $ax^2 + bx + c = 0$  ( $a \neq 0, a, b, c$  being different), then  $(1 + \alpha + \alpha^2)(1 + \beta + \beta^2)$  is  
 (a) zero (b) positive  
 (c) negative (d) None of these
- 8 The minimum value of the sum of real numbers  $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$  and  $a^{10}$   $a > 0$  is  
 (a) 9 (b) 8 (c) 2 (d) 1
- 9 For  $a > 0$ , the roots of the equation  $\log_{ax} a + \log_x a^2 + \log_{a^2x} a^3 = 0$  is given by  
 (a)  $a^{-4/3}$  (b)  $a^{-3/4}$  (c)  $a^{1/2}$  (d)  $a^{-1}$



- 10** If  $a$ ,  $b$  and  $c$  are in AP and if the equations  $(b - c)x^2 + (c - a)x + (a - b) = 0$  and  $2(c + a)x^2 + (b + c)x = 0$  have a common root, then  
 (a)  $a^2, b^2$  and  $c^2$  are in AP (b)  $a^2, c^2$  and  $b^2$  are in AP  
 (c)  $c^2, a^2$  and  $b^2$  are in AP (d) None of these
- 11** If the equations  $x^2 + ax + 12 = 0$ ,  $x^2 + bx + 15 = 0$  and  $x^2 + (a + b)x + 36 = 0$  have a common positive root, then the ordered pair  $(a, b)$  is  
 (a)  $(-6, -7)$  (b)  $(-7, -8)$   
 (c)  $(-6, -8)$  (d)  $(-8, -7)$
- 12** If  $x$  is real, then the maximum and minimum value of the expression  $\frac{x^2 - 3x + 4}{x^2 + 3x + 4}$  will be  
 (a) 2, 1 (b)  $5, \frac{1}{5}$   
 (c)  $7, \frac{1}{7}$  (d) None of these
- 13** If  $a \in R$  and the equation  $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$  (where,  $[x]$  denotes the greatest integer  $\leq x$ ) has no integral solution, then all possible values of  $a$  lie in the interval **→ JEE Mains 2014**  
 (a)  $(-1, 0) \cup (0, 1)$  (b)  $(1, 2)$   
 (c)  $(-2, -1)$  (d)  $(-\infty, -2) \cup (2, \infty)$
- 14** If  $a = \cos \frac{2\pi}{7} + i \sin \frac{2\pi}{7}$ , then the quadratic equation whose roots are  $\alpha = a + a^2 + a^4$  and  $\beta = a^3 + a^5 + a^6$ , is  
 (a)  $x^2 - x + 2 = 0$  (b)  $x^2 + 2x + 2 = 0$   
 (c)  $x^2 + x + 2 = 0$  (d)  $x^2 + x - 2 = 0$
- 15** If  $\alpha$  and  $\beta$  are roots of  $375x^2 - 25x - 2 = 0$  and  $S_n = \alpha^n + \beta^n$ , then  $\lim_{n \rightarrow \infty} \sum_{r=1}^n S_r$  is equal to  
 (a)  $\frac{7}{116}$  (b)  $\frac{1}{12}$   
 (c)  $\frac{29}{358}$  (d) None of these
- 16** If  $S = \{a \in N, 1 \leq a \leq 100\}$  and  $[\tan^2 x] - \tan x - a = 0$  has real roots, where  $[.]$  denotes the greatest integer function, then number of elements in set  $S$  equals  
 (a) 2 (b) 5 (c) 6 (d) 9
- 17** The sum of all real values of  $x$  satisfying the equation  $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$  is **→ JEE Mains 2016**  
 (a) 3 (b) -4 (c) 6 (d) 5
- 18** If  $\lambda$  is an integer and  $\alpha, \beta$  are the roots of  $4x^2 - 16x + \frac{\lambda}{4} = 0$  such that  $1 < \alpha < 2$  and  $2 < \beta < 3$ , then the possible values of  $\lambda$  are  
 (a)  $\{60, 64, 68\}$  (b)  $\{61, 62, 63\}$   
 (c)  $\{49, 50, \dots, 62, 63\}$  (d)  $\{62, 65, 68, 71, 75\}$

- 19** If  $\alpha$  and  $\beta$  are the roots of the equation  $ax^2 + bx + c = 0$ , then the quadratic equation whose roots are  $\frac{\alpha}{1 + \alpha}$  and  $\frac{\beta}{1 + \beta}$  is  
 (a)  $ax^2 - b(1 - x) + c(1 - x)^2 = 0$   
 (b)  $ax^2 - b(x - 1) + c(x - 1)^2 = 0$   
 (c)  $ax^2 + b(1 - x) + c(1 - x)^2 = 0$   
 (d)  $ax^2 + b(x + 1) + c(1 + x)^2 = 0$
- 20** If both the roots of the quadratic equation  $x^2 - 2kx + k^2 + k - 5 = 0$  are less than 5, then  $k$  lies in the interval **→ AIEEE 2005**  
 (a)  $[4, 5]$  (b)  $(-\infty, 4)$  (c)  $(6, \infty)$  (d)  $(5, 6)$
- 21** All the values of  $m$  for which both roots of the equation  $x^2 - 2mx + m^2 - 1 = 0$  are greater than -2 but less than 4 lie in the interval  
 (a)  $m > 3$  (b)  $-1 < m < 3$   
 (c)  $1 < m < 4$  (d)  $-2 < m < 0$
- 22** Let  $\alpha$  and  $\beta$  be real and  $z$  be a complex number. If  $z^2 + \alpha z + \beta = 0$  has two distinct roots on the line  $\text{Re}(z) = 1$ , then it is necessary that **→ AIEEE 2011**  
 (a)  $\beta \in (-1, 0)$  (b)  $|\beta| = 1$  (c)  $\beta \in [1, \infty)$  (d)  $\beta \in (0, 1)$
- 23** The equation  $e^{\sin x} - e^{-\sin x} - 4 = 0$  has **→ AIEEE 2012**  
 (a) infinite number of real roots  
 (b) no real root  
 (c) exactly one real root  
 (d) exactly four real roots
- 24** If  $a, b, c, d$  are positive real numbers such that  $a + \frac{1}{b} = 4, b + \frac{1}{c} = 1, c + \frac{1}{d} = 4$  and  $d + \frac{1}{a} = 1$ , then  
 (a)  $a = c$  and  $b = d$  (b)  $b = d$  but  $a \neq c$   
 (c)  $ab = 1$  and  $cd \neq 1$  (d)  $cd = 1$  and  $ab \neq 1$   
 ( $\omega$  and  $\omega^2$  are complex cube roots of unity)
- 25** Let  $f : R \rightarrow R$  be a continuous function defined by  $f(x) = \frac{1}{e^x + 2e^{-x}}$   
**Statement I**  $f(c) = \frac{1}{3}$ , for some  $c \in R$ .  
**Statement II**  $0 < f(x) \leq \frac{1}{2\sqrt{2}}, \forall x \in R$ . **→ AIEEE 2010**  
 (a) Statement I is false, Statement II is true  
 (b) Statement I is true, Statement II is true.  
 Statement II is a correct explanation of Statement I  
 (c) Statement I is true, Statement II is true;  
 Statement II is not a correct explanation for Statement I  
 (d) Statement I is true, Statement II is false

# ANSWERS

## SESSION 1

- |         |         |         |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (c)  | 2. (a)  | 3. (d)  | 4. (c)  | 5. (a)  | 6. (a)  | 7. (c)  | 8. (a)  | 9. (b)  | 10. (a) |
| 11. (d) | 12. (d) | 13. (c) | 14. (c) | 15. (a) | 16. (d) | 17. (b) | 18. (a) | 19. (c) | 20. (c) |
| 21. (d) | 22. (b) | 23. (b) | 24. (b) | 25. (b) | 26. (c) | 27. (c) | 28. (c) | 29. (a) | 30. (c) |
| 31. (b) | 32. (d) | 33. (c) | 34. (c) | 35. (a) | 36. (b) | 37. (d) | 38. (d) | 39. (c) | 40. (a) |
| 41. (c) | 42. (a) | 43. (d) | 44. (c) | 45. (d) | 46. (b) | 47. (d) | 48. (d) | 49. (a) | 50. (b) |
| 51. (b) | 52. (a) | 53. (a) | 54. (a) | 55. (d) |         |         |         |         |         |

## SESSION 2

- |         |         |         |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (c)  | 2. (d)  | 3. (c)  | 4. (c)  | 5. (a)  | 6. (d)  | 7. (b)  | 8. (b)  | 9. (a)  | 10. (b) |
| 11. (b) | 12. (c) | 13. (a) | 14. (c) | 15. (b) | 16. (d) | 17. (a) | 18. (c) | 19. (c) | 20. (b) |
| 21. (b) | 22. (c) | 23. (b) | 24. (a) | 25. (b) |         |         |         |         |         |

# Hints and Explanations

## SESSION 1

1 We have,

$$\sqrt{3x^2 - 7x - 30} + \sqrt{2x^2 - 7x - 5} = x + 5$$

$$\Rightarrow \sqrt{3x^2 - 7x - 30} = (x + 5) - \sqrt{2x^2 - 7x - 5}$$

On squaring both sides, we get

$$3x^2 - 7x - 30 = x^2 + 25 + 10x + (2x^2 - 7x - 5) - 2(x + 5)\sqrt{2x^2 - 7x - 5}$$

$$\Rightarrow -10x - 50 = -2(x + 5)\sqrt{2x^2 - 7x - 5}$$

$$x + 5 \neq 0, \sqrt{2x^2 - 7x - 5} = 5$$

[ $\because x = -5$  does not satisfy the given equation]

$$\Rightarrow 2x^2 - 7x - 30 = 0$$

$$\therefore x = 6$$

2 Given equation is  $x^2 - 5|x| + 6 = 0$

$$\text{When } x \geq 0, x^2 - 5x + 6 = 0$$

$$\text{and when } x < 0, x^2 + 5x + 6 = 0$$

$$\Rightarrow x^2 - 3x - 2x + 6 = 0; x \geq 0$$

$$\text{and } x^2 + 3x + 2x + 6 = 0; x < 0$$

$$\Rightarrow (x - 3)(x - 2) = 0, x \geq 0$$

$$\text{and } (x + 3)(x + 2) = 0, x < 0$$

$$\therefore x = 3, x = 2 \text{ and } x = -3, x = -2$$

There are four solutions of this equation.

3 Given equation is

$$|2x - 1|^2 - 3|2x - 1| + 2 = 0$$

Let  $|2x - 1| = t$ , then

$$t^2 - 3t + 2 = 0$$

$$\Rightarrow (t - 1)(t - 2) = 0 \Rightarrow t = 1, 2$$

$$\Rightarrow |2x - 1| = 1 \text{ and } |2x - 1| = 2$$

$$\Rightarrow 2x - 1 = \pm 1 \text{ and } 2x - 1 = \pm 2$$

$$\Rightarrow x = 1, 0 \text{ and } x = \frac{3}{2}, -\frac{1}{2}$$

$$4 \because 5 - 2\sqrt{6} = \frac{1}{5 + 2\sqrt{6}}$$

$$\therefore t + \frac{1}{t} = 10,$$

$$\text{where } t = (5 + 2\sqrt{6})^{x^2 - 3} \dots(i)$$

$$\Rightarrow t^2 - 10t + 1 = 0$$

$$\Rightarrow t = 5 \pm 2\sqrt{6}$$

$$\text{or } t = (5 + 2\sqrt{6})^{\pm 1} \dots(ii)$$

From Eqs. (i) and (ii),

$$x^2 - 3 = \pm 1$$

$$\Rightarrow x^2 = 2, 4$$

$$\Rightarrow x = -\sqrt{2}, \sqrt{2}, -2, 2$$

$\therefore$  Required product = 8

5 The given equation is

$$2(1 + i)x^2 - 4(2 - i)x - 5 - 3i = 0$$

$$\Rightarrow x = \frac{4(2 - i) \pm \sqrt{16(2 - i)^2 + 8(1 + i)(5 + 3i)}}{4(1 + i)}$$

$$= -\frac{i}{1 + i} \text{ or } \frac{4 - i}{1 + i} = \frac{-1 - i}{2} \text{ or } \frac{3 - 5i}{2}$$

$$\text{Now, } \left| \frac{-1 - i}{2} \right| = \sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{1}{2}}$$

$$\text{and } \left| \frac{3 - 5i}{2} \right| = \sqrt{\frac{9}{4} + \frac{25}{4}} = \sqrt{\frac{17}{2}}$$

$$\text{Also, } \sqrt{\frac{17}{2}} > \sqrt{\frac{1}{2}}$$

$$\text{Hence, required root is } \frac{3 - 5i}{2}.$$

6 Given equation

$$(1 + 2k)x^2 + (1 - 2k)x + (1 - 2k) = 0$$

If equation is a perfect square then root are equal

$$\text{i.e. } (1 - 2k)^2 - 4(1 + 2k)(1 - 2k) = 0$$

$$\text{i.e. } k = \frac{1}{2}, -\frac{3}{10}$$

Hence, total number of values = 2.

7 Since, roots are real.

$$\therefore \{2(bc + ad)\}^2 = 4(a^2 + b^2)(c^2 + d^2)$$

$$\Rightarrow 4b^2c^2 + 4a^2d^2 + 8abcd = 4a^2c^2 + 4a^2d^2 + 4b^2c^2 + 4b^2d^2$$

$$\Rightarrow 4a^2c^2 + 4b^2d^2 - 8abcd = 0$$

$$\Rightarrow 4(ac - bd)^2 = 0$$

$$\Rightarrow ac = bd$$

$$\Rightarrow \frac{a}{d} = \frac{b}{c}$$

8 Given equation is  $x^2 + ax + 1 = 0$

Since, roots are real

$$\therefore a^2 - 4 \geq 0 \Rightarrow |a| \geq 2$$

Thus, the least value of  $|a|$  is 2.

9 We know that only even prime is 2,

$$\therefore (2)^2 - \lambda(2) + 12 = 0.$$

$$\Rightarrow \lambda = 8 \dots(i)$$

$\therefore x^2 + \lambda x + \mu = 0$  has equal roots.

$$\therefore \lambda^2 - 4\mu = 0 \quad [\because D = 0]$$

$$\Rightarrow (8)^2 - 4\mu = 0 \Rightarrow \mu = 16$$

10 Here,  $D = (3b)^2 - 4(4a)(2c)$

$$= 9b^2 - 32ac = 9(-a - c)^2 - 32ac$$

$$= 9a^2 - 14ac + 9c^2$$

$$= 9c^2 \left[ \left(\frac{a}{c}\right)^2 - \frac{14}{9} \cdot \frac{a}{c} + 1 \right]$$

$$= 9c^2 \left[ \left(\frac{a}{c} - \frac{7}{9}\right)^2 - \frac{49}{81} + 1 \right] > 0$$

Hence, the roots are real and distinct.



**11** For real roots, discriminant,

$$D = b^2 - 4ac \geq 0$$

$$= \cos^2 \beta - 4(\cos \beta - 1)\sin \beta \geq 0$$

$$= \cos^2 \beta + 4(1 - \cos \beta)\sin \beta \geq 0$$

So,  $\sin \beta$  should be  $> 0$ .

$$[\because \cos^2 \beta \geq 0, 1 - \cos \beta \geq 0]$$

$$\Rightarrow \beta \in (0, \pi)$$

**12** Here,  $D = 4b^2 + 12ca < 0$

$$\Rightarrow b^2 + 3ca < 0 \quad \dots(i)$$

$$\Rightarrow ca < 0$$

If  $c > 0$ , then  $a < 0$

$$\text{Also, } \frac{3c}{4} < a + b$$

$$\Rightarrow 3ca > 4a^2 + 4ab$$

$$\Rightarrow b^2 + 3ca > 4a^2 + 4ab + b^2$$

$$= (2a + b)^2 \geq 0 \quad \dots(ii)$$

From (i) and (ii),  $c > 0$ , is not true.

$$\therefore c < 0$$

**13** Since,  $D \geq 0$

$$\therefore b^2 - 4ac \geq 0$$

$$\Rightarrow \left(\frac{c+a}{2}\right)^2 - 4ac \geq 0 \quad [\because 2b = a+c]$$

$$\Rightarrow c^2 - 14ca + a^2 \geq 0$$

$$\Rightarrow \left(\frac{c}{a}\right)^2 - 14\left(\frac{c}{a}\right) + 1 \geq 0$$

$$\Rightarrow \left(\frac{c}{a} - 7\right)^2 \geq 48$$

$$\Rightarrow \left|\frac{c}{a} - 7\right| \geq 4\sqrt{3}$$

**14** Let  $D_1$  and  $D_2$  be the discriminants of given equation, respectively. Then

$$D_1 + D_2 = b^2 - 4ac + d^2 + 4ac$$

$$= b^2 + d^2 > 0$$

So, either  $D_1$  and  $D_2$  are positive or atleast one  $D$ 's is positive.

Therefore,  $P(x) \cdot Q(x) = 0$  has either two or four real roots.

**15** One root =  $\frac{1}{4 + \sqrt{3}} \times \frac{4 - \sqrt{3}}{4 - \sqrt{3}} = \frac{4 - \sqrt{3}}{13}$

$$\therefore \text{Other root} = \frac{4 + \sqrt{3}}{13}$$

$\therefore$  The quadratic equation is

$$x^2 - \left(\frac{4 + \sqrt{3}}{13} + \frac{4 - \sqrt{3}}{13}\right)x$$

$$+ \frac{4 + \sqrt{3}}{13} \cdot \frac{4 - \sqrt{3}}{13} = 0$$

$$\text{or } 13x^2 - 8x + 1 = 0$$

This equation must be identical with  $ax^2 + bx + 1 = 0$ ;

$$\therefore a = 13 \text{ and } b = -8.$$

**16** Sum of roots

$$\frac{-a}{1} = (1-i) + (1+i) \Rightarrow a = -2.$$

[since, non-real complex roots occur in conjugate pairs]

Product of roots,

$$\frac{b}{1} = (1-i)(1+i) \Rightarrow b = 2$$

**17** Let roots be  $\alpha$  and  $\alpha^2$ .

Then,  $\alpha + \alpha^2 = 30$  and  $\alpha^3 = p$

$$\Rightarrow \alpha^2 + \alpha - 30 = 0$$

$$\Rightarrow (\alpha + 6)(\alpha - 5) = 0$$

$$\therefore \alpha = -6, 5$$

$$\Rightarrow p = \alpha^3 = (-6)^3 = -216$$

$$\text{and } p = (5)^3 = 125$$

$$\therefore p = 125 \text{ and } -216$$

**18** Given,  $\frac{x-m}{mx+1} = \frac{x+n}{nx+1}$

$$\Rightarrow x^2(m-n) + 2mnx + (m+n) = 0$$

Roots are  $\alpha, \frac{1}{\alpha}$  respectively, then

$$\alpha \cdot \frac{1}{\alpha} = \frac{m+n}{m-n}$$

$$\Rightarrow m-n = m+n \Rightarrow n = 0.$$

**19** Since,  $\alpha, \alpha^2$  be the roots of the equation

$$x^2 + x + 1 = 0$$

$$\therefore \alpha + \alpha^2 = -1 \quad \dots (i)$$

$$\text{and } \alpha^3 = 1 \quad \dots (ii)$$

Now,  $\alpha^{31} + \alpha^{62} = \alpha^{31}(1 + \alpha^{31})$

$$\Rightarrow \alpha^{31} + \alpha^{62} = \alpha^{30} \cdot \alpha(1 + \alpha^{30} \cdot \alpha)$$

$$\Rightarrow \alpha^{31} + \alpha^{62} = (\alpha^3)^{10} \cdot \alpha(1 + (\alpha^3)^{10} \cdot \alpha)$$

$$\Rightarrow \alpha^{31} + \alpha^{62} = \alpha(1 + \alpha) \quad [\text{from Eq. (ii)}]$$

$$\Rightarrow \alpha^{31} + \alpha^{62} = -1 \quad [\text{from Eq. (i)}]$$

Again,  $\alpha^{31} \cdot \alpha^{62} = \alpha^{93}$

$$\Rightarrow \alpha^{31} \cdot \alpha^{62} = [\alpha^3]^{31} = 1$$

$\therefore$  Required equation is

$$x^2 - (\alpha^{31} + \alpha^{62})x + \alpha^{31} \cdot \alpha^{62} = 0$$

$$\Rightarrow x^2 + x + 1 = 0$$

**20** Since,  $\alpha$  and  $\beta$  are the roots of

$$x^2 - ax + a + b = 0, \text{ then}$$

$$\alpha + \beta = a$$

$$\text{and } \alpha\beta = a + b$$

$$\Rightarrow \alpha^2 + \alpha\beta = a\alpha$$

$$\Rightarrow \alpha^2 - a\alpha = -(a + b)$$

$$\text{and } \alpha\beta + \beta^2 = a\beta$$

$$\Rightarrow \beta^2 - a\beta = -(a + b)$$

$$\therefore \frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} + \frac{2}{a + b}$$

$$= \frac{1}{-(a+b)} + \frac{1}{-(a+b)} + \frac{2}{(a+b)} = 0$$

**21** Let  $\alpha$  and  $\beta$  be the roots of equation

$$x^2 - (a-2)x - a - 1 = 0$$

Then,  $\alpha + \beta = a - 2$  and  $\alpha\beta = -a - 1$

Now,  $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$\Rightarrow \alpha^2 + \beta^2 = (a-2)^2 + 2(a+1)$$

$$\Rightarrow \alpha^2 + \beta^2 = a^2 - 2a + 6$$

$$\Rightarrow \alpha^2 + \beta^2 = (a-1)^2 + 5$$

The value of  $\alpha^2 + \beta^2$  will be least, if

$$a - 1 = 0$$

$$\Rightarrow a = 1$$

**22** Since,  $\alpha$  and  $\beta$  are the roots of the

equation

$$2x^2 + 2(a+b)x + a^2 + b^2 = 0$$

$$\therefore (\alpha + \beta)^2 = (a+b)^2 \text{ and } \alpha\beta = \frac{a^2 + b^2}{2}$$

$$\text{Now, } (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

$$= (a+b)^2 - 4\left(\frac{a^2 + b^2}{2}\right)$$

$$= -(a-b)^2$$

Now, the required equation whose

roots are  $(\alpha + \beta)^2$  and  $(\alpha - \beta)^2$  is

$$x^2 - \{(\alpha + \beta)^2 + (\alpha - \beta)^2\}x$$

$$+ (\alpha + \beta)^2(\alpha - \beta)^2 = 0$$

$$\Rightarrow x^2 - \{(a+b)^2 - (a-b)^2\}x$$

$$- (a+b)^2(a-b)^2 = 0$$

$$\Rightarrow x^2 - 4abx - (a^2 - b^2)^2 = 0$$

**23** The given equation is

$$x^2 - 2x\cos\phi + 1 = 0$$

$$\therefore x = \frac{2\cos\phi \pm \sqrt{4\cos^2\phi - 4}}{2}$$

$$= \cos\phi \pm i\sin\phi$$

Let  $\alpha = \cos\phi + i\sin\phi$ , then

$$\beta = \cos\phi - i\sin\phi$$

$$\therefore \alpha^n + \beta^n = (\cos\phi + i\sin\phi)^n$$

$$+ (\cos\phi - i\sin\phi)^n$$

$$= 2\cos n\phi$$

and  $\alpha^n\beta^n = (\cos n\phi + i\sin n\phi)$

$$\cdot (\cos n\phi - i\sin n\phi)$$

$$= \cos^2 n\phi + \sin^2 n\phi = 1$$

$\therefore$  Required equation is

$$x^2 - 2x\cos n\phi + 1 = 0$$

**24** Given equation is

$$(5 + \sqrt{2})x^2 - (4 + \sqrt{5})x + 8 + 2\sqrt{5} = 0$$

Let  $x_1$  and  $x_2$  are the roots of the equation, then

$$x_1 + x_2 = \frac{4 + \sqrt{5}}{5 + \sqrt{2}} \quad \dots (i)$$

$$\text{and } x_1 x_2 = \frac{8 + 2\sqrt{5}}{5 + \sqrt{2}} = \frac{2(4 + \sqrt{5})}{5 + \sqrt{2}}$$

$$= 2(x_1 + x_2) \quad \dots (ii)$$

Clearly, harmonic mean

$$= \frac{2x_1 x_2}{x_1 + x_2} = \frac{4(x_1 + x_2)}{(x_1 + x_2)} = 4$$

[from Eq. (ii)]

**25** Let  $\alpha, \beta$  and  $\alpha', \beta'$  be the roots of the given equations, respectively.

$$\therefore \alpha + \beta = -\frac{\mu}{\lambda}, \alpha\beta = \frac{\nu}{\lambda} \quad \dots (i)$$

and  $\alpha' = \omega$  and  $\beta' = \omega^2$

$$\therefore \frac{\alpha}{\beta} = \frac{\alpha'}{\beta'} \quad [\text{given}]$$

$$\Rightarrow \frac{\alpha}{\beta} = \frac{\omega}{\omega^2} \Rightarrow \beta = \alpha\omega$$

From Eq. (i),

$$\alpha + \alpha\omega = -\frac{\mu}{\lambda}, \alpha^2\omega = \frac{\nu}{\lambda}$$

$$\Rightarrow -\alpha\omega^2 = -\frac{\mu}{\lambda}, \alpha^2\omega = \frac{\nu}{\lambda}$$

$$[\because -\omega^2 = 1 + \omega]$$

$$\Rightarrow \frac{\mu^2}{\lambda^2} = \frac{\nu}{\lambda} \Rightarrow \mu^2 = \lambda\nu$$

**26** Simplified form of given equation is

$$(2x + p + q)r = (x + p)(x + q)$$

$$\Rightarrow x^2 + (p + q - 2r)x - (p + q)r + pq = 0$$

Since, sum of roots = 0

$$\Rightarrow -(p + q - 2r) = 0$$

$$\Rightarrow r = \frac{p + q}{2}$$

and product of roots

$$= -(p + q)r + pq$$

$$= -\frac{(p + q)^2}{2} + pq$$

$$= -\frac{1}{2}(p^2 + q^2)$$

**27** Clearly, sum of roots,

$$\frac{k+1}{k} + \frac{k+2}{k+1} = -\frac{b}{a} \quad \dots (i)$$

and product of roots,

$$\frac{k+1}{k} \times \frac{k+2}{k+1} = \frac{c}{a}$$

$$\Rightarrow \frac{k+2}{k} = \frac{c}{a}$$

$$\Rightarrow \frac{2}{k} = \frac{c}{a} - 1 = \frac{c-a}{a} \Rightarrow k = \frac{2a}{c-a}$$

On putting the value of  $k$  in Eq. (i), we get

$$\frac{c+a}{2a} + \frac{2c}{c+a} = -\frac{b}{a}$$

$$\Rightarrow (c+a)^2 + 4ac = -2b(a+c)$$

$$\Rightarrow (a+c)^2 + 2b(a+c) = -4ac$$

$$\Rightarrow (a+c)^2 + 2b(a+c) + b^2 = b^2 - 4ac$$

$$\Rightarrow (a+b+c)^2 = b^2 - 4ac$$

**28** Since,  $\alpha$  and  $\beta$  are the roots of the equation  $ax^2 + bx + c = 0$

$$\therefore \alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

Now, sum of roots =  $|\alpha| + |\beta|$

$$= -\alpha - \beta \quad (\because \beta < \alpha < 0)$$

$$= -\left(-\frac{b}{a}\right) = \frac{b}{a} \quad (\because |\alpha| + |\beta| > 0)$$

and product of roots =  $|\alpha||\beta| = \frac{c}{a}$

Hence, required equation is

$$x^2 - \frac{b}{a}|x| + \frac{c}{a} = 0$$

$$\Rightarrow |a|x^2 - |b|x + |c| = 0$$

**29** Since,  $\alpha$  and  $\beta$  are the roots of the equation  $x^2 + px + q = 0$  therefore  $\alpha + \beta = -p$  and  $\alpha\beta = q$

Now,  $(\omega\alpha + \omega^2\beta)(\omega^2\alpha + \omega\beta)$

$$= \alpha^2 + \beta^2 + (\omega^4 + \omega^2)\alpha\beta \quad (\because \omega^3 = 1)$$

$$= \alpha^2 + \beta^2 - \alpha\beta \quad (\because \omega + \omega^2 = -1)$$

$$= (\alpha + \beta)^2 - 3\alpha\beta = p^2 - 3q$$

Also,  $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$

$$= \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta} = \frac{p(3q - p^2)}{q}$$

$\therefore$  The given expression

$$= \frac{(p^2 - 3q)}{p(3q - p^2)} = -\frac{q}{p}$$

**30** Clearly,  $\alpha + \beta = -p$  and  $\alpha\beta = \frac{3p}{4}$

Also,  $(\alpha - \beta)^2 = 10$

$$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 10$$

$$\Rightarrow p^2 - 3p = 10$$

$$\Rightarrow (p+2)(p-5) = 0$$

$$\therefore p = -2, 5$$

**31** Let the quadratic equation be  $ax^2 + bx + c = 0$  and its roots are  $\alpha$  and  $\beta$ .

Sachin made a mistake in writing down constant term.

$\therefore$  Sum of roots is correct i.e.  $\alpha + \beta = 7$

Rahul made mistake in writing down coefficient of  $x$ .

$\therefore$  Product of roots is correct.

i.e.  $\alpha \cdot \beta = 6$

$\Rightarrow$  Correct quadratic equation is

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0.$$

$$\Rightarrow x^2 - 7x + 6 = 0 \text{ having roots } 1 \text{ and } 6.$$

**32** Since,  $\alpha + \beta = -1, \alpha\beta = 2,$

$$\gamma + \delta = -3, \text{ and } \gamma\delta = 4$$

$$\therefore (\alpha + \gamma)(\alpha + \delta)(\beta + \gamma)(\beta + \delta)$$

$$= (\alpha^2 - 3\alpha + 4)(\beta^2 - 3\beta + 4)$$

$$= 4 - 3\alpha\beta^2 + 4\beta^2 - 3\alpha^2\beta + 9\alpha\beta$$

$$- 12\beta + 4\alpha^2 - 12\alpha + 16$$

$$= 4 - 3(2)\beta + 4\beta^2 + 4\alpha^2$$

$$- 3(2)\alpha + 9(2) - 12(\beta + \alpha) + 16$$

$$= 4 - 6\beta + 4(\alpha^2 + \beta^2)$$

$$- 6\alpha + 18 + 12 + 16$$

$$= 50 + 6 + 4[(\alpha + \beta)^2 - 2\alpha\beta]$$

$$= 56 - 12 = 44$$

**33** Given,  $R = \frac{\pi}{2} \Rightarrow P + Q = \frac{\pi}{2}$

$$\Rightarrow \frac{P}{2} + \frac{Q}{2} = \frac{\pi}{4}$$

$$\Rightarrow 1 = \tan \frac{\pi}{4} = \frac{\tan \frac{P}{2} + \tan \frac{Q}{2}}{1 - \tan \frac{P}{2} \tan \frac{Q}{2}}$$

$$\Rightarrow 1 = \frac{-\frac{b}{a}}{1 - \frac{c}{a}} = \frac{b}{c-a} \Rightarrow a + b = c$$

**34** Given,  $(6k + 2)x^2 + rx + 3k - 1 = 0$   
and  $(12k + 4)x^2 + px + 6k - 2 = 0$

For both common roots,

$$\frac{6k + 2}{12k + 4} = \frac{r}{p} = \frac{3k - 1}{6k - 2}$$

$$\Rightarrow \frac{r}{p} = \frac{1}{2} \Rightarrow 2r - p = 0$$

**35** Given equations are

$$x^2 + 2x + 3 = 0 \quad \dots (i)$$

$$\text{and } ax^2 + bx + c = 0 \quad \dots (ii)$$

Since, Eq. (i) has imaginary roots.

So, Eq. (ii) will also have both roots same as Eq. (i)

Thus,  $\frac{a}{1} = \frac{b}{2} = \frac{c}{3}$

Hence,  $a:b:c$  is  $1:2:3$ .

**36**  $\alpha, \beta$  be the roots of  $ax^2 + bx + c = 0$

$$\therefore \alpha + \beta = -b/a, \alpha\beta = c/a$$

Now roots are  $\alpha - 1, \beta - 1$

Their sum,  $\alpha + \beta - 2 = (-b/a) - 2$

$$= -8/2 = -4$$

Their product,

$$(\alpha - 1)(\beta - 1) = \alpha\beta - (\alpha + \beta) + 1$$

$$= c/a + b/a + 1 = 1$$

$\therefore b/a = 2$  i.e.  $b = 2a$

also  $c + b = 0 \Rightarrow b = -c$ .

**37**  $\min f(x) = -\frac{D}{4a} = -\frac{4b^2 - 8c^2}{4a}$

$$= -(b^2 - 2c^2)$$

(upward parabola)

$$\max g(x) = -\frac{D}{4a} = \frac{4c^2 + 4b^2}{4a}$$

$$= b^2 + c^2$$

(downward parabola)

Now,  $2c^2 - b^2 > b^2 + c^2$

$$\Rightarrow c^2 > 2b^2 \Rightarrow |c| > \sqrt{2}|b|$$

**38** We have,

$$(x - a_1)^2 + (x - a_2)^2 + \dots + (x - a_n)^2$$

$$= n x^2 - 2x(a_1 + a_2 + \dots + a_n)$$

$$+ (a_1^2 + a_2^2 + \dots + a_n^2)$$

So, it attains its minimum value at

$$x = \frac{2(a_1 + a_2 + \dots + a_n)}{2n}$$

$$= \frac{a_1 + a_2 + \dots + a_n}{n} \quad \left[ \text{using } : x = \frac{-b}{2a} \right]$$

**39** Given  $bx^2 + cx + a = 0$  has imaginary roots.

$$\Rightarrow c^2 - 4ab < 0 \Rightarrow c^2 < 4ab \quad \dots (i)$$

Let  $f(x) = 3b^2x^2 + 6bcx + 2c^2$

Here,  $3b^2 > 0$

So, the given expression has a minimum value.

$$\therefore \text{Minimum value} = \frac{-D}{4a}$$

$$= \frac{4ac - b^2}{4a} = \frac{4(3b^2)(2c^2) - 36b^2c^2}{4(3b^2)}$$

$$= -\frac{12b^2c^2}{12b^2} = -c^2 > -4ab$$

[from Eq. (i)]

- 40** According to given condition,  
 $4a^2 - 4(10 - 3a) < 0$   
 $\Rightarrow a^2 + 3a - 10 < 0$   
 $\Rightarrow (a + 5)(a - 2) < 0$   
 $\Rightarrow -5 < a < 2.$

- 41** We have,  $ax - 1 + \frac{1}{x} \geq 0$   
 $\Rightarrow \frac{ax^2 - x + 1}{x} \geq 0$   
 $\Rightarrow ax^2 - x + 1 \geq 0$  as  $x > 0$   
 It will hold if  $a > 0$  and  $D \leq 0$   
 $a > 0$  and  $1 - 4a \leq 0 \Rightarrow a \geq \frac{1}{4}$

Therefore, the minimum value of  $a$  is  $\frac{1}{4}$ .

- 42** Let  $f(x) = -3 + x - x^2$ .  
 Then,  $f(x) < 0$  for all  $x$ , because coeff. of  $x^2 < 0$  and disc.  $< 0$ .  
 Thus, LHS of the given equation is always positive whereas the RHS is always less than zero. Hence, the given equation has no solution.

- 43** Let  $f(x) = ax^2 + bx + c$ .  
 Then,  $k$  lies between  $\alpha$  and  $\beta$ , if a  $f(k) < 0$   
 $\Rightarrow a(ak^2 + bk + c) < 0$   
 $\Rightarrow a^2k^2 + abk + ac < 0.$

- 44** We have,  $|2x - 3| < |x + 5|$   
 $\Rightarrow |2x - 3| - |x + 5| < 0$   
 $\Rightarrow \begin{cases} 3 - 2x + x + 5 < 0, x \leq -5 \\ 3 - 2x - x - 5 < 0, -5 < x \leq \frac{3}{2} \\ 2x - 3 - x - 5 < 0, x > \frac{3}{2} \end{cases}$   
 $\Rightarrow \begin{cases} x > 8, x \leq -5 \\ x > -\frac{2}{3}, -5 < x \leq \frac{3}{2} \\ x < 8, x > \frac{3}{2} \end{cases}$   
 $\Rightarrow x \in \left(-\frac{2}{3}, \frac{3}{2}\right] \cup \left(\frac{3}{2}, 8\right)$   
 $\Rightarrow x \in \left(-\frac{2}{3}, 8\right)$

- 45**  $\frac{x - 5}{x^2 + 5x - 14} > 0$   
 $\Rightarrow \frac{(x + 7)(x - 2)(x - 5)}{(x - 2)^2(x + 7)^2} > 0$

$\Rightarrow x \in (-7, 2) \cup (5, \infty)$   
 So, the least integral value  $\alpha$  of  $x$  is  $-6$ , which satisfy the equation  
 $\alpha^2 + 5\alpha - 6 = 0$

- 46** Given,  $\frac{|x - 2| - 1}{|x - 2| - 2} \leq 0$

Let  $|x - 2| = k$   
 Then, given equation,  
 $\frac{k - 1}{k - 2} \leq 0 \Rightarrow \frac{(k - 1)(k - 2)}{(k - 2)^2} \leq 0$   
 $\Rightarrow (k - 1)(k - 2) \leq 0 \Rightarrow 1 \leq k \leq 2$   
 $\Rightarrow 1 \leq |x - 2| \leq 2$

**Case I** When  $1 \leq |x - 2|$   
 $\Rightarrow |x - 2| \geq 1$   
 $\Rightarrow x - 2 \geq 1$  or  $x - 2 \leq -1$   
 $\Rightarrow x \geq 3$  and  $x \leq 1$  ... (i)

**Case II** When  $|x - 2| \leq 2$   
 $\Rightarrow -2 \leq x - 2 \leq 2$   
 $\Rightarrow -2 + 2 \leq x \leq 2 + 2$   
 $\Rightarrow 0 \leq x \leq 4$  ... (ii)  
 From (i) and (ii),  $x \in [0, 1] \cup [3, 4]$

- 47**  $\frac{x + 2}{x^2 + 1} - \frac{1}{2} > 0$   
 $\Rightarrow \frac{-x^2 - 1 + 2x + 4}{2(x^2 + 1)} > 0$   
 $\Rightarrow \frac{3 + 2x - x^2}{2(x^2 + 1)} > 0$

Since, denominator is positive  
 $\therefore 3 + 2x - x^2 > 0$   
 $\Rightarrow -1 < x < 3$   
 $\Rightarrow x = 0, 1, 2$

- 48** Let  $a_1, a_2, \dots, a_n$  be  $n$  positive integers such that  $a_1 a_2 \dots a_n = 1$ .  
 Using AM  $\geq$  GM, we have  
 $\Rightarrow \frac{a_1 + a_2 + \dots + a_n}{n} \geq (a_1 a_2 \dots a_n)^{1/n}$   
 $\Rightarrow a_1 + a_2 + \dots + a_n \geq n.$

- 49** Since,  $AM \geq GM$   
 $\therefore \frac{bcx + cay + abz}{3} \geq (a^2b^2c^2 \cdot xyz)^{1/3}$   
 $\Rightarrow bcx + cay + abz \geq 3abc$  [ $\because xyz = abc$ ]

- 50** We know that,  $AM > GM$   
 $\therefore \frac{a + b + c}{3} > (abc)^{1/3}$  ... (i)

and  $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} > \left(\frac{1}{abc}\right)^{1/3}$   
 $\Rightarrow \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) > \frac{3}{(abc)^{1/3}}$  ... (ii)

From (i) and (ii), we get  
 $\left(\frac{a + b + c}{3}\right) \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) > \frac{3}{(abc)^{1/3}} \cdot (abc)^{1/3}$   
 $\Rightarrow (a + b + c) \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) > 9$

- 51** Using  $GM \leq AM$ , we have  
 $\{(1 - a)(1 - b)(1 - c)\}^{1/3}$   
 $\leq \frac{1 - a + 1 - b + 1 - c}{3}$   
 $\Rightarrow (1 - a)(1 - b)(1 - c) \leq \frac{8}{27}$   
 Hence, the greatest value is  $\frac{8}{27}$ .

- 52** Using  $AM \geq GM$ , we have  
 $\frac{(a + b) + (c + d)}{2} \geq \{(a + b)(c + d)\}^{1/2}$   
 $\Rightarrow \frac{2}{2} \geq M^{1/2} \Rightarrow M \leq 1$   
 As  $a, b, c, d > 0$ .  
 Therefore,  $M = (a + b)(c + d) > 0$ .  
 Hence,  $0 \leq M \leq 1$ .

- 53**  $\log_2(x^2 - 3x + 18) < 4$   
 $\Rightarrow x^2 - 3x + 18 < 16$   
 $(\because \log_a b < c \Leftrightarrow b < a^c, \text{ if } a > 1)$   
 $\Rightarrow x^2 - 3x + 2 < 0$   
 $\Rightarrow (x - 1)(x - 2) < 0 \Rightarrow x \in (1, 2)$

- 54.** Clearly,  $x - 1 > 0 \Rightarrow x > 1$   
 and  $\log_{0.3}(x - 1) > \log_{(0.3)^2}(x - 1)$   
 $\Rightarrow \log_{0.3}(x - 1) > \frac{1}{2} \log_{0.3}(x - 1)$   
 $\Rightarrow \log_{0.3}(x - 1) > 0 \Rightarrow x < 2$   
 Hence,  $x \in (1, 2)$

- 55** By definition of  $\log x$ ,  $x > 0$  and  
 $\left(\frac{x + 5}{1 - 3x}\right) > 0$   
 $\Rightarrow \frac{(x + 5)(1 - 3x)}{(1 - 3x)^2} > 0$   
 $\Rightarrow (x + 5)(1 - 3x) > 0$   
 $\Rightarrow (x + 5)(3x - 1) < 0$   
 $\Rightarrow -5 < x < 1/3$   
 As  $x > 0$ ,  $0 < x < 1/3$   
 $\therefore \frac{x + 5}{1 - 3x} < 1 \Rightarrow x < -1$   
 This does not satisfy  $0 < x < 1/3$ .  
 Hence, there is no solution.

## SESSION 2

**1** We have

$$2\sqrt{x - 3} + \sqrt{x}(\sqrt{x - 6}) + 6 = 0$$

Let  $\sqrt{x - 3} = y$

$$\Rightarrow \sqrt{x} = y + 3$$

$$\therefore 2|y| + (y + 3)(y - 3) + 6 = 0$$

$$\Rightarrow 2|y| + y^2 - 3 = 0$$

$$\Rightarrow |y| + 2|y| - 3 = 0$$

$$\Rightarrow (|y| + 3)(|y| - 1) = 0$$

$$\Rightarrow |y| \neq -3 \Rightarrow |y| = 1$$

$$\begin{aligned} \Rightarrow y &= \pm 1 \\ \Rightarrow \sqrt{x} - 3 &= \pm 1 \\ \Rightarrow \sqrt{x} &= 4, 2 \\ \Rightarrow x &= 16, 4 \end{aligned}$$

2 We have,  $2^{x+2} \cdot 3^{3x/(x-1)} = 9$

Taking log on both sides, we get

$$(x+2)\log 2 + \frac{3x}{(x-1)}\log 3 = 2\log 3$$

$$\Rightarrow (x^2 + x - 2)\log 2 + 3x\log 3 = 2(x-1)\log 3$$

$$\Rightarrow x^2 \log 2 + (\log 2 + \log 3)x - 2\log 2 + 2\log 3 = 0$$

$$- (\log 2 + \log 3) \pm \sqrt{\{(\log 2 + \log 3)^2 - 4\log 2(-2\log 2 + 2\log 3)\}}$$

$$\therefore x = \frac{2\log 2 \pm \sqrt{\{(\log 2 + \log 3)^2 - 4\log 2(-2\log 2 + 2\log 3)\}}}{2\log 2}$$

$$= \frac{-(\log 2 + \log 3) \pm \sqrt{\{(3\log 2)^2 - 6\log 2\log 3 + (\log 3)^2\}}}{2\log 2}$$

$$= \frac{-(\log 2 + \log 3) \pm (3\log 2 - \log 3)}{2\log 2}$$

$$\therefore x = -2, 1 - \frac{\log 3}{\log 2}$$

3 Given,  $\alpha$  and  $\beta$  are the roots of the equation  $x^2 - 6x - 2 = 0$

$$\therefore a_n = \alpha^n - \beta^n, n \geq 1 \Rightarrow a_{10} = \alpha^{10} - \beta^{10}$$

$$a_8 = \alpha^8 - \beta^8 \text{ and } a_6 = \alpha^6 - \beta^6$$

Now, consider

$\frac{a_{10} - 2a_6}{2a_8}$	$[\because \alpha \text{ and } \beta \text{ are the roots of}$
$= \frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{2(\alpha^8 - \beta^8)}$	$x^2 - 6x - 2 = 0$
$= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^8 - \beta^8)}$	or $x^2 = 6x + 2$
$= \frac{\alpha^8 \cdot 6\alpha - \beta^8 \cdot 6\beta}{2(\alpha^8 - \beta^8)}$	$\Rightarrow \alpha^2 = 6\alpha + 2$
$= \frac{6\alpha^9 - 6\beta^9}{2(\alpha^8 - \beta^8)} = \frac{6}{2} = 3$	$\Rightarrow \alpha^2 - 2 = 6\alpha$
	and $\beta^2 = 6\beta + 2$
	$\Rightarrow \beta^2 - 2 = 6\beta$

4 We have,  $x^2 - 5x + 1 = 0$

$$x + \frac{1}{x} = 5 \Rightarrow x^2 + \frac{1}{x^2} = 5^2 - 2 = 23$$

$$\text{and } x^3 + \frac{1}{x^3} = 5^3 - 3 \times 5 = 110$$

$$\text{Now, } \left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right)$$

$$= 23 \times 110 = 2530$$

$$\Rightarrow x^5 + \frac{1}{x^5} = 2530 - \left(x + \frac{1}{x}\right) = 2525$$

5 Given equation can be rewritten as  $3x^2 - (a+c+2b+2d)x + ac + 2bd = 0$

Now, discriminant,  $D$

$$= (a+c+2b+2d)^2 - 4 \cdot 3(ac+2bd)$$

$$= \{(a+2d) + (c+2b)\}^2 - 12(ac+2bd)$$

$$= \{(a+2d) - (c+2b)\}^2 + 4(a+2d)(c+2b) - 12(ac+2bd)$$

$$= \{(a+2d) - (c+2b)\}^2 - 8ac + 8ab + 8dc - 8bd$$

$$= \{(a+2d) - (c+2b)\}^2 + 8(c-b)(d-a)$$

Which is +ve, since  $a < b < c < d$ . Hence, roots are real and distinct.

6 Since,  $\alpha$  and  $\beta$  are roots of

$$px^2 + qx + r = 0, p \neq 0$$

$$\therefore \alpha + \beta = \frac{-q}{p}, \alpha\beta = \frac{r}{p}$$

Since,  $p, q$  and  $r$  are in AP.

$$\therefore 2q = p + r$$

Also,  $\frac{1}{\alpha} + \frac{1}{\beta} = 4 \Rightarrow \frac{\alpha + \beta}{\alpha\beta} = 4$

$$\Rightarrow \alpha + \beta = 4\alpha\beta \Rightarrow \frac{-q}{p} = \frac{4r}{p}$$

$$\therefore 2q = p + r$$

$$\Rightarrow 2(-4r) = p + r \Rightarrow p = -9r$$

$$\therefore \alpha + \beta = \frac{-q}{p} = \frac{4r}{-9r} = -\frac{4}{9}$$

$$\text{and } \alpha\beta = \frac{r}{p} = \frac{r}{-9r} = -\frac{1}{9}$$

Now, consider

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

$$= \frac{16}{81} + \frac{4}{9} = \frac{16 + 36}{81}$$

$$\Rightarrow (\alpha - \beta)^2 = \frac{52}{81}$$

$$\Rightarrow |\alpha - \beta| = \frac{2\sqrt{13}}{9}$$

7  $\alpha + \beta = -b/a$  and  $\alpha\beta = c/a$

Now,  $(1 + \alpha + \alpha^2)(1 + \beta + \beta^2)$

$$= 1 + (\alpha + \beta) + (\alpha^2 + \beta^2) + \alpha\beta + \alpha\beta(\alpha + \beta) + (\alpha\beta)^2$$

$$= 1 + (\alpha + \beta) + (\alpha + \beta)^2 - \alpha\beta + \alpha\beta(\alpha + \beta) + (\alpha\beta)^2$$

$$= 1 - \frac{b}{a} + \frac{b^2}{a^2} - \frac{c}{a} + \left(\frac{c}{a}\right)\left(-\frac{b}{a}\right) + \frac{c^2}{a^2}$$

$$= \frac{(a^2 + b^2 + c^2 - ab - bc - ca)}{a^2}$$

$$= \frac{[(a-b)^2 + (b-c)^2 + (c-a)^2]}{2a^2}$$

which is positive.

8 Using AM  $\geq$  GM

$$a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3}$$

$$\therefore \frac{1 + a^8 + a^{10}}{8}$$

$$\geq (a^{-5} \cdot a^{-4} \cdot a^{-3} \cdot a^{-3} \cdot a^{-3} \cdot 1 \cdot a^8 \cdot a^{10})^{1/8}$$

$$\Rightarrow a^{-5} + a^{-4} + 3a^{-3} + 1 + a^8 + a^{10} \geq 8 \cdot 1$$

Hence, minimum value is 8.

9 We have,

$$\frac{\log_a a}{\log_a a + \log_a x} + \frac{2\log_a a}{\log_a x} + \frac{3\log_a a}{2\log_a a + \log_a x} = 0$$

$$\Rightarrow \frac{1}{1+t} + \frac{2}{t} + \frac{3}{2+t} = 0$$

(let  $\log_a x = t$ )

$$\Rightarrow \frac{2t + t^2 + 2t^2 + 6t + 4 + 3t^2 + 3t}{t(1+t)(2+t)} = 0$$

$$\Rightarrow 6t^2 + 11t + 4 = 0$$

$$\Rightarrow 6t^2 + 8t + 3t + 4 = 0$$

$$\Rightarrow (2t+1)(3t+4) = 0$$

$$\Rightarrow t = -\frac{1}{2}, -\frac{4}{3}$$

$$\Rightarrow \log_a x = -\frac{1}{2}, -\frac{4}{3}$$

$$\therefore x = a^{-1/2}, a^{-4/3}$$

10 Since,  $x = 1$  is a root of first equation. If  $\alpha$  is another root of first equation, then

$$\alpha \times 1 = \alpha = \frac{a-b}{b-c}$$

(Product of roots)

$$= \frac{2a-2b}{2b-2c} = \frac{2a-(a+c)}{a+c-2c} = 1$$

So, the roots of first equation are 1 and 1.

Since, the equations have a common root, 1 is also a root of second equation.

$$\Rightarrow 2(c+a) + b + c = 0$$

$$\Rightarrow 2(2b) + b + c = 0$$

[since,  $a, b$  and  $c$  are in AP]

$$\Rightarrow c = -5b$$

Also,  $a + c = 2b \Rightarrow a = 2b - c$

$$= 2b + 5b = 7b$$

$$\therefore a^2 = 49b^2, c^2 = 25b^2$$

Hence,  $a^2, c^2$  and  $b^2$  are in AP.

11 On adding first and second equations, we get

$$2x^2 + (a+b)x + 27 = 0$$

On subtracting above equation from given third equation, we get

$$x^2 - 9 = 0 \Rightarrow x = 3, -3$$

Thus, common positive root is 3.

$$\text{Now, } (3)^2 + 3a + 12 = 0 \Rightarrow a = -7$$

$$\text{and } 9 + 3b + 15 = 0 \Rightarrow b = -8$$

Hence, the order pair  $(a, b)$  is  $(-7, -8)$ .

12 Let  $y = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$

$$\Rightarrow (y-1)x^2 + 3(y+1)x + 4(y-1) = 0$$

$\therefore x$  is real.

$$\therefore D \geq 0$$

$$\Rightarrow 9(y+1)^2 - 16(y-1)^2 \geq 0$$

$$\Rightarrow -7y^2 + 50y - 7 \geq 0$$

$$\Rightarrow 7y^2 - 50y + 7 \leq 0$$

$$\Rightarrow (y-7)(7y-1) \leq 0$$

$$\Rightarrow y \leq 7 \text{ and } y \geq \frac{1}{7}$$

$$\Rightarrow \frac{1}{7} \leq y \leq 7$$

Hence, maximum value is 7 and minimum value is  $\frac{1}{7}$ .

- 13** Here,  $a \in R$  and equation is  $-3\{x - [x]\}^2 + 2\{x - [x]\} + a^2 = 0$

Let  $t = x - [x]$ , then  $-3t^2 + 2t + a^2 = 0$

$$\Rightarrow t = \frac{1 \pm \sqrt{1 + 3a^2}}{3}$$

$\therefore t = x - [x] = \{x\}$  [fractional part]

$\therefore 0 \leq t \leq 1$

$$\Rightarrow 0 \leq \frac{1 \pm \sqrt{1 + 3a^2}}{3} < 1$$

[ $\because 0 \leq \{x\} < 1$ ]

But  $1 - \sqrt{1 + 3a^2} < 0$  therefore

$$0 \leq \frac{1 + \sqrt{1 + 3a^2}}{3} < 1$$

$$\Rightarrow \sqrt{1 + 3a^2} < 2$$

$$\Rightarrow 1 + 3a^2 < 4 \Rightarrow a^2 - 1 < 0$$

$$\Rightarrow (a + 1)(a - 1) < 0$$

$$\Rightarrow a \in (-1, 1)$$

For no integral solution we consider the interval  $(-1, 0) \cup (0, 1)$ .

- 14** Given,  $a = \cos \frac{2\pi}{7} + i \sin \frac{2\pi}{7}$

$$\therefore a^7 = \cos 2\pi + i \sin 2\pi = 1$$

[ $\because e^{i\theta} = \cos \theta + i \sin \theta$ ]

Also,  $\alpha = a + a^2 + a^4$ ,

$\beta = a^3 + a^5 + a^6$

Then, the sum of roots,

$$S = \alpha + \beta = a + a^2 + a^3 + a^4 + a^5 + a^6$$

$$\Rightarrow S = \frac{a(1 - a^6)}{1 - a} = \frac{a - a^7}{1 - a}$$

$$= \frac{a - 1}{1 - a} = -1 \quad [\because a^7 = 1]$$

and product of the roots,

$$P = \alpha\beta = (a + a^2 + a^4)(a^3 + a^5 + a^6)$$

$$= a^4 + a^5 + 1 + a^6 + 1 + a^2 + 1 + a + a^3 \quad [\because a^7 = 1]$$

$$= 3 + (a + a^2 + a^3 + a^4 + a^5 + a^6)$$

$$= 3 - 1 = 2$$

Hence, the required quadratic equation is  $x^2 + x + 2 = 0$

- 15** Since,  $\alpha$  and  $\beta$  are the roots of

$$375x^2 - 25x - 2 = 0.$$

$$\therefore \alpha + \beta = \frac{25}{375} = \frac{1}{15}$$

and  $\alpha\beta = -\frac{2}{375}$

Now, consider

$$\begin{aligned} \lim_{n \rightarrow \infty} \sum_{r=1}^n S_r &= \lim_{n \rightarrow \infty} \sum_{r=1}^n (\alpha^r + \beta^r) \\ &= (\alpha + \alpha^2 + \alpha^3 + \dots \infty) \\ &\quad + (\beta + \beta^2 + \beta^3 + \dots \infty) \\ &= \frac{\alpha}{1 - \alpha} + \frac{\beta}{1 - \beta} = \frac{\alpha - \alpha\beta + \beta - \alpha\beta}{(1 - \alpha)(1 - \beta)} \\ &= \frac{\alpha + \beta - 2\alpha\beta}{1 - (\alpha + \beta) + \alpha\beta} \\ &= \frac{1}{15} + \frac{4}{375} \\ &= \frac{1 - \frac{1}{15} - \frac{2}{375}}{1 - \frac{1}{15} - \frac{2}{375}} \\ &= \frac{25 + 4}{375 - 25 - 2} \\ &= \frac{29}{348} = \frac{1}{12} \end{aligned}$$

- 16** Here,  $[\tan^2 x] = \text{integer}$

and  $a = \text{integer}$

So,  $\tan x$  is also an integer.

Then,  $\tan^2 x - \tan x - a = 0$

$$\Rightarrow a = \tan x (\tan x - 1) = I(I - 1)$$

= Product of two

consecutive integers

$$\therefore a = 2, 6, 12, 20, 30, 42, 56, 72, 90$$

Hence, set  $S$  has 9 elements.

- 17** Given,  $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$

Clearly, this is possible when

I.  $x^2 + 4x - 60 = 0$  and

$$x^2 - 5x + 5 \neq 0$$

or

II.  $x^2 - 5x + 5 = 1$

or

III.  $x^2 - 5x + 5 = -1$  and

$$x^2 + 4x - 60 = \text{Even integer}$$

**Case I** When  $x^2 + 4x - 60 = 0$

$$\Rightarrow x^2 + 10x - 6x - 60 = 0$$

$$\Rightarrow x(x + 10) - 6(x + 10) = 0$$

$$\Rightarrow (x + 10)(x - 6) = 0$$

$$\Rightarrow x = -10 \text{ or } x = 6$$

Note that, for these two values of  $x$ ,  $x^2 - 5x + 5 \neq 0$

**Case II** When  $x^2 - 5x + 5 = 1$

$$\Rightarrow x^2 - 5x + 4 = 0$$

$$\Rightarrow x^2 - 4x - x + 4 = 0$$

$$\Rightarrow (x - 4)(x - 1) = 0$$

$$\Rightarrow x = 4 \text{ or } x = 1$$

**Case III** When  $x^2 - 5x + 5 = -1$

$$\Rightarrow x^2 - 5x + 6 = 0$$

$$\Rightarrow x^2 - 2x - 3x + 6 = 0$$

$$\Rightarrow x(x - 2) - 3(x - 2) = 0$$

$$\Rightarrow (x - 2)(x - 3) = 0$$

$$\Rightarrow x = 2 \text{ or } x = 3$$

Now, when  $x = 2$ ,

$$x^2 + 4x - 60 = 4 + 8 - 60 = -48, \text{ which}$$

is an even integer.

When  $x = 3$ ,

$$x^2 + 4x - 60 = 9 + 12 - 60 = -39, \text{ which}$$

is not an even integer.

Thus, in this case, we get  $x = 2$

Hence, the sum of all real values of

$$x = -10 + 6 + 4 + 1 + 2 = 3$$

- 18** Given,  $4x^2 - 16x + \frac{\lambda}{4} = 0$

$$\therefore x = \frac{16 \pm \sqrt{(256 - 4\lambda)}}{8}$$

$$= \frac{8 \pm \sqrt{(64 - \lambda)}}{4}$$

$$\Rightarrow \alpha, \beta = 2 \pm \frac{\sqrt{(64 - \lambda)}}{4}$$

Here,  $64 - \lambda > 0$

$$\therefore \lambda < 64$$

Also,  $1 < \alpha < 2$  and  $2 < \beta < 3$

$$\therefore 1 < 2 - \frac{\sqrt{64 - \lambda}}{4} < 2$$

and  $2 < 2 + \frac{\sqrt{64 - \lambda}}{4} < 3$

$$\Rightarrow -1 < -\frac{\sqrt{64 - \lambda}}{4} < 0$$

and  $0 < \frac{\sqrt{64 - \lambda}}{4} < 1$

$$\Rightarrow 1 > \frac{\sqrt{64 - \lambda}}{4} > 0$$

and  $0 < \frac{\sqrt{64 - \lambda}}{4} < 1$

i.e.  $0 < \frac{\sqrt{64 - \lambda}}{4} < 1$

$$\Rightarrow 0 < \sqrt{64 - \lambda} < 4$$

$$\Rightarrow 0 < 64 - \lambda < 16 \Rightarrow \lambda > 48$$

or  $48 < \lambda < 64$

$$\therefore \lambda = \{49, 50, 51, 52, \dots, 63\}$$

- 19** Since, roots of  $ax^2 + bx + c = 0$  are  $\alpha$  and  $\beta$ . Hence, roots of  $cx^2 + bx + a = 0$ , will

be  $\frac{1}{\alpha}$  and  $\frac{1}{\beta}$ . Now, if we replace  $x$  by

$x - 1$ , then roots of

$$c(x - 1)^2 + b(x - 1) + a = 0 \text{ will be } 1 + \frac{1}{\alpha}$$

and  $1 + \frac{1}{\beta}$ . Now, again replace  $x$  by  $\frac{1}{x}$ ,

we will get  $c(1 - x)^2 + b(1 - x) + ax^2 = 0$ ,

whose roots are  $\frac{\alpha}{1 + \alpha}$  and  $\frac{\beta}{1 + \beta}$ .

- 20** Let  $f(x) = x^2 - 2kx + k^2 + k - 5$

Since, both roots are less than 5.

$$\therefore D \geq 0, -\frac{b}{2a} < 5 \text{ and } f(5) > 0$$

Here,  $D = 4k^2 - 4(k^2 + k - 5)$

$$= -4k + 20 \geq 0$$

$$\Rightarrow k \leq 5 \quad \dots (i)$$

$$-\frac{b}{2a} < 5 \Rightarrow k < 5 \quad \dots \text{(ii)}$$

and  $f(5) > 0$

$$\Rightarrow 25 - 10k + k^2 + k - 5 > 0$$

$$\Rightarrow k^2 - 9k + 20 > 0$$

$$\Rightarrow (k-5)(k-4) > 0$$

$$\Rightarrow k < 4 \text{ and } k > 5 \quad \dots \text{(iii)}$$

From (i), (ii) and (iii), we get  $k < 4$

**21** Since, both roots of equation  $x^2 - 2mx + m^2 - 1 = 0$  are greater than  $-2$  but less than  $4$ .

$$\therefore D \geq 0, -2 < -\frac{b}{2a} < 4,$$

$$f(4) > 0 \text{ and } f(-2) > 0$$

Now,  $D \geq 0$

$$\Rightarrow 4m^2 - 4m^2 + 4 \geq 0$$

$$\Rightarrow 4 > 0 \Rightarrow m \in R \quad \dots \text{(i)}$$

$$-2 < -\frac{b}{2a} < 4$$

$$\Rightarrow -2 < \left(\frac{2m}{2 \cdot 1}\right) < 4$$

$$\Rightarrow -2 < m < 4 \quad \dots \text{(ii)}$$

$$f(4) > 0$$

$$\Rightarrow 16 - 8m + m^2 - 1 > 0$$

$$\Rightarrow m^2 - 8m + 15 > 0$$

$$\Rightarrow (m-3)(m-5) > 0$$

$$\Rightarrow -\infty < m < 3 \text{ and } 5 < m < \infty \dots \text{(iii)}$$

and  $f(-2) > 0$

$$\Rightarrow 4 + 4m + m^2 - 1 > 0$$

$$\Rightarrow m^2 + 4m + 3 > 0$$

$$(m+3)(m+1) > 0$$

$$\Rightarrow -\infty < m < -3 \text{ and } -1 < m < \infty \quad \dots \text{(iv)}$$

From (i), (ii), (iii) and (iv), we get  $m$  lie between  $-1$  and  $3$ .

**22** Let  $z = x + iy$ , given  $\text{Re}(z) = 1$

$$\therefore x = 1 \Rightarrow z = 1 + iy$$

Since, the complex roots are conjugate to each other.

So,  $z = 1 + iy$  and  $1 - iy$  are two roots of  $z^2 + \alpha z + \beta = 0$ .

$$\therefore \text{Product of roots} = \beta$$

$$\Rightarrow (1 + iy)(1 - iy) = \beta$$

$$\therefore \beta = 1 + y^2 \geq 1 \Rightarrow \beta \in [1, \infty)$$

**23** Given equation is

$$e^{\sin x} - e^{-\sin x} = 4 \Rightarrow e^{\sin x} - \frac{1}{e^{\sin x}} = 4$$

Let  $e^{\sin x} = t$ , then  $t - \frac{1}{t} = 4$

$$\Rightarrow t^2 - 1 - 4t = 0 \Rightarrow t^2 - 4t - 1 = 0$$

$$\Rightarrow t = \frac{4 \pm \sqrt{16+4}}{2}$$

$$t = 2 \pm \sqrt{5} \Rightarrow e^{\sin x} = 2 \pm \sqrt{5}$$

But  $-1 \leq \sin x \leq 1 \Rightarrow e^{-1} \leq e^{\sin x} \leq e^1$

$$\Rightarrow e^{\sin x} \in \left[\frac{1}{e}, e\right]$$

Also,  $0 < e < 2 + \sqrt{5}$   
Hence, given equation has no solution.

**24** Using  $AM > GM$ , we have

$$a + \frac{1}{b} > 2\sqrt{\frac{a}{b}}, \quad b + \frac{1}{c} > 2\sqrt{\frac{b}{c}}$$

$$c + \frac{1}{d} > 2\sqrt{\frac{c}{d}} \text{ and } d + \frac{1}{a} > 2\sqrt{\frac{d}{a}}$$

$$\left(a + \frac{1}{b}\right)\left(b + \frac{1}{c}\right)\left(c + \frac{1}{d}\right)\left(d + \frac{1}{a}\right) > 16$$

But,  $\left(a + \frac{1}{b}\right)\left(b + \frac{1}{c}\right)\left(c + \frac{1}{d}\right)$

$$\left(d + \frac{1}{a}\right) = 4 \times 1 \times 4 \times 1 = 16$$

$$\therefore a = \frac{1}{b}, b = \frac{1}{c}, c = \frac{1}{d} \text{ and } d = \frac{1}{a}$$

$$\Rightarrow a = \frac{1}{b} = 2, b = \frac{1}{c} = \frac{1}{2}, c = \frac{1}{d} = 2$$

and  $d = \frac{1}{a} = \frac{1}{2}$

$$\Rightarrow a = 2, b = \frac{1}{2}, c = 2 \text{ and } d = \frac{1}{2}$$

$$\Rightarrow a = c \text{ and } b = d$$

**25** We have,  $f(x) = \frac{1}{e^x + \frac{2}{e^x}}$

Using  $AM \geq GM$ , we get

$$\frac{e^x + \frac{2}{e^x}}{2} \geq \left(e^x \cdot \frac{2}{e^x}\right)^{1/2}, \text{ as } e^x > 0$$

$$\Rightarrow e^x + \frac{2}{e^x} \geq 2\sqrt{2}$$

$$\Rightarrow 0 < \frac{1}{e^x + \frac{2}{e^x}} \leq \frac{1}{2\sqrt{2}}$$

$$\therefore 0 < f(x) \leq \frac{1}{2\sqrt{2}}, \forall x \in R$$

Statement II is true and Statement I is also true as for some 'c'.

$$\Rightarrow f(c) = \frac{1}{3} \quad [\text{for } c = 0]$$

which lies between  $0$  and  $\frac{1}{2\sqrt{2}}$ .

So, Statement II is correct explanation of Statement I.